EFFICIENCY OF T-STATISTICS FOR TESTING TWO NORMALS SAMPLES

P.V. KRISHNA IYER*, B.N. SINGH†

AND

M.S. HOLLA‡

Introduction

A sequence of two alternatives *A* and *B* is obtained by arranging the observations of two samples *A* and *B* in ascending order. Let the sample *A* consist of *m* observations and the sample *B* of *n* observations. A number of *statistics* arising from such sequences has been considered by various authors [Mann and Whitney (1947), Kruskal (1952), Mood (1954), Stuart (1952), Iyer and Singh (1955)]. A few of these *statistics* relevant to this paper are :

(*i*) the number of runs of *A's* and *B's*,

(*ii*) the number of *AB* or *BA* transitions between adjoining or successive observations,

(*iii*) the number of *A's* preceding *B's* in the sequence,

(*iv*) the sum of the ranks of *A's* or *B's*

(*v*) the number of *AB* or *BA* transitions between pairs separated by $(r-2)$ observations or less in the sequence.

The statistic mentioned in (*iii*) is called Mann and Whitney's *U* statistic and the statistic given by (*iv*) is commonly known as Wilcoxon's *statistic*. There is a close relation between the latter two statistics. Taking S_n to be the sum of the ranks of the observations of *B* and *U* as the number of *AB* transitions in the sequence, it can be easily seen that

$$S_n = U + \frac{n(n+1)}{2}.$$

The statistic mentioned in (*v*) has been called as T_r and for $r=m+n$, it reduces to *U statistic*.

A series of investigations carried out by Iyer and Singh (1955), Singh (1966) and Iyer and Ray (1964) show (*i*) that the distribution of T_r tends to the normal form for values of $m+n(=S$, say) greater than thirty, (*ii*) that the power of the

* University of Queensland, Brisbane.

† Indian Standards Institution, New Delhi

‡ I.A.R.S., New Delhi.

statistic for testing randomness of a binomial sequence is maximum at some point for $r \leqslant s/2$ when the alternative is a Markov chain such that the measure of dependence $\delta = p_1 - p_2$, between successive observations is positive, p_1 and p_2, being the conditional probabilities $P(A/A)$ and $P(A/B)$ of the sequence and (*iii*) that the power of the *statistic* for testing two samples is maximum at some point for $r \leqslant s/2$ and hence more than that for the *U statistic* in certain cases when the form of the distribution of the parent population is not known. It may incidentally be mentioned that Wetherill (1960) has established that the Wilcoxon's test is a little more robust than the *t*-test for testing differences in population variances and is much more sensitive to skewness and kurtosis. For testing location of two samples which belong to identical non-normal populations, Wilcoxon's test is to be preferred although *t*-test is insensitive to small departures from normality.

As regards T_r, the fact that T_r is more powerful than U under certain circumstances makes it desirable to know as to how this test would behave when the two samples belong to the same normal population. We shall therefore examine the power of T_r as compared to U for two samples belonging to a normal population.

2. Preliminaries and Notations

Let $x_1, x_2, \ldots\ldots x_m$ and $y_1, y_2 \ldots\ldots y_n$ be two random samples of sizes m and n from two populations with distribution functions F and G and density functions f and g respectively. Pool together the two samples and arrange them in ascending or descending order of magnitude. The ordered arrangement of the two samples gives a binominal sequence of m x's and n y's. The *T-statistics* for this sequence may be defined as follows:

$$T_2 = \sum_{i=1}^{s-1} z_{i,\,i+1}$$ where $z_{i,\,i+1}=1$ if the ith and $(i+1)$th observations of the sequence are x and y respectively; $z_{i,\,i+1}=0$ otherwise. ...(1)

$$T_3 = T_2 + \sum_{i=1}^{s-2} z_{i,\,i+2}$$ where $z_{i,\,i+2}=1$ if the ith and $(i+2)$-th observations are x and y respectively; $z_{i,\,i+2}=0$ otherwise ...(2)

....

....

$$T_k = T_{k-1} + \sum_{i=1}^{s-k+1} z_{i,\,i+k-1}$$ where $z_{i,\,i+k-1}=1$ if ith and $(i+k-1)$th observations are x and y respectively and zero otherwise ...(3)

It may be noted that T_{m+n} reduces to the U-statistic and T_2 gives the number of AB transitions between successive observations.

3. Expected values and Variances of T on the Basis of Ordered Statistics

Consider two samples of size m and n. Let $z_1, z_2, \ldots\ldots z_r$ where $s=m+n$, represent the ordered values of these two samples. Also let $f(z)$ and $g(z)$ denote the density functions of the two samples. The probability of obtaining an AB transition between successive observations z_i and z_{i+1}, *i.e.* z_i belonging to A and z_{i+1} to B is given by

$$\rho_{i1}=\left[\frac{m f(z_i)}{m f(z_i)+ng(z_i)}\right]\left[\frac{ng(z_{i+1})}{(m-1) f(z_{i+1})+ng(z_{i+1})}\right]$$

Similarly the probability for an AB transition between the i-th and $(i+k)$th observation is

$$\rho_{ik}=\left[\frac{m f(z_i)}{m f(z_i)+ng(z_i)}\right]\left[\frac{ng(z_{i+k})}{(m-1)(z_{i+k})+ng(z_{i+k})}\right] \qquad \ldots(4)$$

It can be easily seen that

$$E(T_2)=E\sum_{i=1}^{s-1}(z_i, i+1)=\sum_{i=1}^{s-1}E(z_i, i+1)=(s-1)\rho_1 \qquad \ldots(5)$$

where ρ_k is the average of ρ_{ik} for different values of i

$V(T_2)=E(T_2)+2$(the sum of the expectations for two AB transitions between i-th and $(i+1)$th observations and jth and $(j+1)$th observations in the sequence) - $[E(T_2)]^2$ ...(6)

The expected value of T_3 and its variance are given by

$$E(T_3)=(s-1)\rho_1+(s-2)\rho_2 \qquad \ldots(7)$$

$V(T_3)=E(T_3)+2$(the sum of the expectations for *two* AB transitions like ABB or AAB from three consecutive observations) $+2$ (the sum of the expectations for two AB transitions from four observations such that each of the transitions AB is *not* separated by *more* than one observation) $-\{E(T_3)\}^2$ ...(8)

$$E(T_k)=\sum_{l=2}^{k}(s-\rho+1)\,p_{l-1} \qquad \ldots(9)$$

$V(T_k)=E(T_k)+2$ (the sum of the expectations for two AB transitions arising from three observations like A......B......B and A......A......B where the number of observations between any two observations of the configuration is k or less than k)

$+2$ (the sum of the expectations two transisions like A...B and A...B from four observations such that each of the A....B transitions is separated by $1, 2, \ldots. (k-1)$ observations).

$-[E(T_k)]^2$ ...(10)

To evaluate $E(T_k)$ and $V(T_k)$ we use the following results :

$$(i)\ \rho_{ik}=\left[\frac{mf(z_i)}{mf(z_i)+ng(z_i)}\right]\left[\frac{ng(z_{i+k})}{(m-1)f(z_{i+k})+ng(z_{i+k})}\right]$$

$$\simeq\frac{mn}{s(s-1)}\left[1+\sum_{t=1}^{u}\delta_t\left\{\frac{m-1}{s-1}\frac{f_t'(z_{i+k})}{f(z_{i+k})}-\frac{n}{s}\frac{f_t'(z_i)}{f(z_i)}\right\}\right]\quad\ldots(11)$$

where $f(z)$ and $g(z)$ stand for $f(z, \theta)$, $g(z, \theta)$, θ being a vector involving u parameters $\theta_1, \theta_2, \ldots \theta_k$. It has been assumed that $g(z, \theta)=f(z, \theta+\triangle\theta)$. Also $f_t'(z_i)$ stands for the differential coefficient of $f(z, \theta)$ with respect to θ_t.

The probability ρ_{icd} for an $A\ A\ B$ transition between the observations z_i, z_{i+c} and z_{i+d} is given by

$$(ii)\ \rho_{icd}=\frac{m^{(2)}n\ f(z_i)\ f(z_i+c)\ g(z_i+d)}{[mf(z_i)+ng(z_i)][(m-1)f(z_{i+c})+ng(z_{i+c})][(m-2)f(z_{i+d})+ng(z_{i+d})]}\quad\ldots(12)$$

$$\cong\frac{m^{(2)}n}{s(s-1)(s-2)}\left[1+\sum_{t=1}^{u}\delta_t\left\{\frac{m-2}{s-2}\frac{f_t'(z_{i+d})}{f(z_{i+d})}-\frac{n}{s-1}\frac{f_t'(z_{i+c})}{f(z_{i+c})}-\frac{n}{s}\frac{f_t'(z_i)}{f(z_i)}\right\}\right]\quad\ldots(13)$$

(*iii*) The probability for an ABB transition from z_i, z_{i+c} and z_{i+d} reduces to

$$\rho_{icd}\simeq\frac{mn^{(2)}}{s^{(3)}}\left[1+\sum_{t=1}^{u}\Delta\theta_t\left\{\frac{m-1}{s-1}\frac{f_t'(z_{i+c})}{f(z_{i+c})}+\frac{m-1}{s-2}\frac{f_t'(z_{i+d})}{f(z_{i+d})}-\frac{n}{s}\frac{f_t'(z_i)}{f'(z_i)}\right\}\right]\quad\ldots(14)$$

(*iv*) The probability for an $AB\ AB$ transition form z_i, z_{i+c}, z_{i+d} and z_{i+e} is obtained from

$$\rho_{icde}\simeq\frac{m^{(2)}n^{(2)}}{s^{(4)}}\left[1+\sum_{t=1}^{u}\Delta\theta_t\left\{\frac{m-2}{s-3}\frac{f_t'(z_{i+c})}{f(z_{i+c})}+\frac{m-1}{s-1}\frac{f_t'(z_{i+c})}{f(z_{i+c})}-\frac{n-1}{s-2}\frac{f_t'(z_{i+d})}{f(z_{i+d})}-\frac{n}{s}\frac{f_t'(z_i)}{f(z_i)}\right\}\right]\quad\ldots(15)$$

The above four results can be easily established by using Taylor's expansion omitting second and higher order terms in $\triangle\theta$.

when f and g refer to the normal distributions with mean μ and $\mu+\delta$ and variance σ_0^2, since

$\dfrac{f'(z)}{f^2(z)}=\dfrac{(z-\mu)}{\sigma_0^2}$, the above four results reduce to

$$\rho_{ik}\simeq\frac{mn}{s^{(2)}}\left[1+\frac{\delta}{\sigma_0^2}\left\{\mu\left(\frac{n}{s}-\frac{m-1}{s-1}\right)+\frac{m-1}{s-1}\ z_{i+k}-\frac{n}{s}z_i\right\}\right]\quad\ldots(16)$$

$$\rho_{icd} \simeq \frac{m^{(2)}n}{s^{(2)}}\left[1+\frac{\delta}{\sigma_0^2}\left\{\mu\left(\frac{n}{s}+\frac{n}{s-1}-\frac{m-2}{s-2}\right)+\frac{m-2}{s-2}z_{i+d} - \frac{n}{s-1}z_{i+c}-\frac{n}{s}z_i\right\}\right] \quad ...(17)$$

$$\rho'_{icd} \simeq \frac{mn^{(2)}}{s^{(3)}}\left[1+\frac{\delta}{\sigma_0^2}\left\{\mu\left(\frac{n}{s}-\frac{m-1}{s-2}-\frac{m-1}{s-1}\right)+\frac{m-1}{s-1}z_{i+c} + \frac{m-1}{s-2}z_{i+d}-\frac{n}{s}z_i\right\}\right] \quad ...(18)$$

$$\rho_{icde} \simeq \frac{m^{(2)}n^{(2)}}{s^{(4)}}\left[1+\frac{\delta}{\sigma_0^2}\left\{\mu\left(\frac{n}{s}+\frac{n-1}{s-2}-\frac{m-1}{s-1}-\frac{m-2}{s-3}\right) +\frac{m-2}{s-3}z_{i+c}+\frac{m-1}{s-1}z_{i+c}-\frac{n-1}{s-2}z_{i+d}-\frac{n}{s}z_i\right\}\right]$$

Using these values we can obtain the values of

$E(T_2)$, $V(T_2)$, $E(T_3)$, $V(T_3)$, etc.

$$E(T_2) \simeq \frac{mn}{s^{(2)}}\left[(s-1)+\frac{\delta}{\sigma_0^2}\left\{(s-1)\mu\left(\frac{n}{s}-\frac{m-1}{s-1}\right) + \frac{m-1}{s-1}\sum_{i=1}^{s-1} z_{i+1}-\frac{n}{s}\sum_{i=1}^{s-1} z_i\right\}\right] \quad ...(19)$$

$$V(T_2) \simeq \frac{m^2n^2}{s^2(s-1)}+\frac{\delta}{\sigma_0^2}\left[\frac{mn}{s(s-1)}\left\{\left(\frac{m}{s}-\frac{m-1}{s-1}\right)\mu+\frac{m-1}{s-1}\sum_{i=1}^{s-1} z_{i+1}-\frac{n}{s}\sum_{i=1}^{s-1} z_i\right\} \left\{1-\frac{2mn}{s}\right\}+\frac{2m^{(2)}n^{(2)}}{s^{(4)}}\mu\,(s-2)\,(s-3)\times\ldots\ \frac{(n-1)}{s-2}+\frac{n}{s}-\frac{m-2}{s-3}-\frac{m-1}{s-1}\Big) + \ldots\ldots-\frac{2n}{s}\{(s-3)z_1+(s-4)z_2+\ldots z_{s-3})\}\right] \quad ...(20)$$

$$E(T_r) \simeq \frac{mn}{s^{(2)}}\sum_{k=1}^{r-1}\left[(s-k)+\frac{\delta}{\sigma_0^2}\left\{\frac{m-1}{s-1}\sum_{i=1}^{s-k} z_{i+k}-\frac{n}{s}\sum_{i=1}^{s-k} z_i -(s-k)\,\mu\left(\frac{m-1}{s-1}-\frac{n}{s}\right)\right\}\right] \quad ...(21)$$

If, however, $\mu=0$ and $\sigma_0^2=1$, (21) reduces to

$$E(T_k)=\frac{mn}{s^{(2)}}\sum_{k=1}^{r-1}\left[(s-k)+\delta\left\{\frac{m-1}{s-1}\sum_{i=1}^{s-k} z_{i+k}-\frac{n}{s}\sum_{i=1}^{s-k} z_i\right\}\right] \quad ...(22)$$

The expression for $V(T_k)$ is very cumbersome and therefore has not been evaluated. However, for finite values of k, it can be evaluated with the aid of the expressions given in (11), (12), (13) and (14).

When $F=G$, *i.e.*, when $\delta=0$, the expected values of T_k and $V(T_k)$ reduce to the following : $\left(r\leqslant\frac{s}{2}\right)$

$$E(T_r)=\frac{(r-1)(2s-r)}{2}\,\frac{mn}{s^{(2)}} \quad ...(23)$$

$$V(T_k)=\frac{1}{6}\,(r-1)\,[6s(r-1)-r\,(4r-5)]\,\frac{mn}{s^{(2)}}+\frac{r-1}{12}$$

$$\left[\,3(r-1)(2s-r)^2-2(4r-5)-(6s-5r)\times\frac{m^{(2)}n^{(2)}}{s^{(4)}}-\left[\frac{(r-1)\,(2s-r)}{2}\,\frac{mn}{s^{(2)}}\right]^2\right. \quad ...(24)$$

If $\qquad r=(s-K)>\frac{s}{2}$, then

$$E(T_{s-k})=\frac{(s-K-1)(s+K)}{2}\,\frac{mn}{s^{(2)}} \quad ...(25)$$

$$V(T_{s-K})=\frac{1}{6}\,[3(s-K-1)\,(s+K)+2(s-K-1)^{(2)}\,(s+2K)]\,\frac{mn}{s^{(2)}}$$

$$+\left[\frac{(s-K-1)^2\,(s+K)}{4}-\frac{1}{2}\,(s-K-1)(s+K)-\frac{1}{3}\,s^{(3)}-\frac{2}{3}\,(s-K-1)^{(2)}\,(s+2K)\right.$$

$$\left.+\left(\frac{2}{3}\right)K(k-1)\,\right]\,\frac{m^{(2)}n^{(2)}}{s^{(4)}}-\left[\frac{(s-K-1)\,(s+K)}{2}\,\frac{mn}{s^{(2)}}\right]^2 \quad ...(26)$$

4. Comparative Efficiency of T_r for Testing two Normal Samples

The relative efficiency of the *statistics* T_r for binomial and Markovian sequences has been studied by Iyer and Singh (1955) and Iyer and Ray (1966). These studies show that for binomial squences the power of T_r is maximum for $r\leqslant\frac{s}{2}$. For comparing two samples also the power is maximum for $r\leqslant\frac{s}{2}$, provided no information is available regarding the form of the parent population. We shall now examine the behaviour of these statistics with normal samples for translation alternatives. For this purpose we shall first establish the criteria to be used for finding the relative power of T_r.

Assuming T_n and $T_n{}^*$ to be two estimates of $\mu(\theta)$ and $\mu^*(\theta)$ which are functions of the parameter θ with variances $\sigma_n{}^2(T_n)$ and $\sigma_n{}^{*2}\,(T_n)$ it can be seen that $E(T_n)=\mu_n(\theta)$ and $E(T_n{}^*)=\mu_n{}^*(\theta)$ and the variance of θ as estimated from $\mu_n(\theta)$ and $\mu_n{}^*(\theta)$ can be approximated to $\dfrac{\sigma_n{}^2}{\left\{\dfrac{d\mu_n(\theta)}{d\theta}\right\}^2}$ and $\dfrac{\sigma_n{}^{*2}}{\left\{\dfrac{d\mu(\theta)}{d\theta}\right\}^2}$ respectively. Therefore the efficiency of T_n and $T_n{}^*$ can be taken to be inversely proportional to the variances of θ obtained from these *statistics*. This argument leads immediately to the result of Mood (1954) and Pitman (1948) for relative efficiency (R.E.) of two statistics. It may, however, be noted that Mood obtains this expression by considering the change in the power of the *statistics* for the alternative on the assumption there is no change in the variance of the *statistic* under the null and non-null hypothesis. But this assumption is not justified. The variance of T_r under H_0 and H_1 differ to some extent. Consequently in examining the R.E. of T_r for varying values of r it is necessary to consider the change in the power of these *statistics* by making allowance for the deviation in the variances under H_0 and H_1. Taking the significance level of the test to be α, we note that

$$\alpha = 1 - \int_{-k}^{k} \phi(t)\, dt \quad \text{where } \phi(t) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-t^2}{2}\right) \qquad .. (27)$$

Let $P(\theta)$ be the power function of any *statistic* T. Then

$$P(\theta) = P\{\, |\, T - \mu(\theta)\, | > k\sigma_0(\theta)\}$$

$$\cong 1 - \frac{1}{\sqrt{2\pi}\,\sigma_0(\theta)} \int_{\mu_0((\theta) - k\sigma_0(\theta)}^{\mu_0(\theta) + k\sigma_0(\theta)} \exp\left[-\left\{\frac{y - \mu_1(\theta)}{2\sigma_0^2}\right\}^2\right] dy$$

$$\cong \alpha + \frac{k\phi(k)}{\sigma_0^2(\theta)} \left\{\frac{d\mu_0(\theta)}{d\theta}\right\}^2 (\theta - \theta_0)^2$$

$$\cong \alpha + \frac{k\phi(k)}{\sigma_0^2}\, \eta^2 \qquad ...(28)$$

where η^2 is the square of the change in T under the alternate hypothesis. The above expression is based on the assumption that the variance of T under H_0 and H_1 is the same. If allowance is made for the change in variance, then

$$P(\theta) \simeq \alpha + K\phi(K)\left[\zeta + \frac{\eta^2}{\sigma_0^2}\right] \qquad ...(29)$$

where $\qquad \eta = E(T_{k1}) - E(T_{k0})$ and $\sigma_1^2 = \sigma_0^2(1+\zeta)$

It is obvious from (28) and (29) that $\frac{\eta^2}{\sigma_0^2}$ and $\left(\zeta + \frac{\eta^2}{\sigma_0^2}\right)$ show that relative change in power under NB_1.

Using the criteria η^2/σ_0^2 and $(\zeta + \eta^2/\sigma_0^2)$, the R.E. of the *statistics* T_r for different values of r for normal translation alternatives was evaluated for two samples of size fifteen each from the normal population $N(0, 1)$ and $N(0{\cdot}1, 1)$ as detailed below. The average deviates of the order *statistics* for a sample of thirty observations from the standard normal population were noted from Fisher and Yates Table. The values of $E(T_r)$ and $V(T_r)$ for $r = 2$ to 30 were calculated with the help of (19) and (24) and (26). The values of η^2/σ_0^2 for the different T_r' were also evaluated. Further values of $(\zeta + \eta^2/\sigma_0^2)$ were calculated using (11), (12), (13), (14) and (15) given earlier. The values of η^2/σ_0^2 and $(\zeta + \eta^2/\sigma_0^2)$ have been shown in Fig. 1. This figure shows that if we assume the variances of T_r under the two hypotheses to be the same, the maximum power is attained for T_{30}. This corresponds to Mann and Whitney's *U statistic.* However, the power reaches almost near the maximum value for T_{11}. The power of T_2 is minimum. As T_2 is closely related to the number of runs of *A's* or *B's* in the sequence, we may conclude that the usual run test is not of much use for testing two samples. When we consider the increase in the variance for the alternative, the power is maximum for T_{15} and is definitely more than that of T_{30}. Therefore our previous finding that T_r is more efficient for $r \leqslant s/2$ holds good for translation alternatives of samples from normal population also.

5. Monté Carlo Studies

The studies described above were supplemented by Monte Carlo Studies. For this purpose 1035 sets of random samples, each set consisting of two samples each of 15 observations were drawn from a normal population with zero mean and unit standard deviation $N(0, 1)$. The two samples in each set were pooled and arranged in ascending order and the values of T_r were computed. For the alternative, one of the samples was taken from $N(0{\cdot}1, 1)$ and the other from $N(0, 1)$ and the values of T_r were again separately determined with the help of *IBM* 1620. The expected values and the variances of T_r for null and non-null hypothesis were computed. Comparing the values of the variances it was seen that the variance for the non-null case was in general a little less than that for the null hypothesis.

The power was evaluated by both the procedures considered earlier. The power component obtained is shown in Fig. 2. In the present investigations we note that the variance for *AB* transitions under the non-null hypothesis is in general less than that for the null hypothesis while the expected value is more. With some analysis it can be seen that the variance for the sum of *AB* and *BA* transitions is four times that for *AB* transitions under the null hypothesis. The covariance between *AB* and *BA* transitions will be approximately equal to the variance under the null hypothesis. Since the variance for non-null hypothesis of *AB* transition is less than that for the null hypothesis, the variance for *BA* Transition under non-null conditions will be more than that for null case and will approximately satisfy the condition that variance of *AB* plus the variance for *BA* transitions will be equal to twice the variance for *AB* or *BA* transitions under the null hypothesis. It, therefore, follows that for *BA* transitions the increase in variance is almost the same as the decrease observed for *AB* transitions. Following this, the power given in Fig. 2 is that for *BA* joins.

It would be seen from Fig. 2 that the power observed from η_i^2/σ_0^2 for $k=15$ is only slightly less than the maximum attained for $k=30$. If the change in the variance for non-null hypothesis is considered then the power is maximum for *BA* transitions for $k=9$. The other *statistics* T_8, T_{10}, T_{11} and T_{15} are good competitors. The *U* statistic does not seem to be the optimum statistic under these conditions.

6. Summary

A binomial sequence is obtained by ordering a pair of samples A and B of sizes m and n. A number of *statistics* arising from this sequence has been considered by various authors for testing the location parameters. Among them, the test based on the sum of the ranks of one of the samples has been found to be nearly as powerful as the 't'-test for testing two normal samples. This test is also equivalent to the *U-statistic* of Mann and Whitney. The *U-statistic* gives the number of AB or BA transitions occurring between any two observations of the sequence. Instead of taking all the transitions, one may consider transitions between observations separated by $(r-2)$ or less in the sequence. It has been found that their *statistics* is nearly as efficient as the U for $r \leqslant \frac{(m+n)}{2}$, when it is assumed that the variance of T_r under the null and non-null hypothesis are the same. This assumption is not in general justifiable because the variance under the two hypothesis are not the same. When this is so, T_r appears to be far more powerful than U for $r \leqslant \frac{(m+n)}{2}$ for testing *even* two normal samples. The superiority of the test for samples from populations about which no information is available regarding the form of the distribution has already been established earlier. Thus on the whole T_r, for $r \leqslant \frac{(m+n)}{2}$ appears to be a more efficient statistic for testing two samples than most of the other tests recommended. The comparative efficiency of this test as compared to Dixon's c^2-test (1940) is not known. This would need further investigations.

REFERENCES

Dixon, W.J. (1940) : A criterion for testing the hypothesis that two samples are from the same population. Ann. Math. Stat, 11, 199-204.

Iyer, P.V.K. and Singh B.N. (1955) : On the probability distributions arising from a sequence of observations and their applications. J. Ind. Soc. Agri. Stat., 7, 127-168.

Iyer, P.V.K. and Ray, D. (1964) : Distribution of Mann and Whitney and allied statistics for two-state Markov Chain. Def. Sci. Jour., 14, 213.

Kruskal, W.H. (1952). Nonparametric tests in several sample problems. Ann. Math. Stat. 23, 525-540.

Mann, H.B. and Whitney, D.R. (1947): On a test whether one of two random variables is stochastically larger than the other. Ann. Math. Stat. 18, 50-60.

Mood, A.M. (1954). On the asymptotic efficiency of certain non-parametric two sample tests. Ann. Math. Stat., 25, 514-522.

Pitman, E.J.G. (1948) : Lecture Notes on Non-parametric Statistical Inference, University of North Carolina.

Singh, B.N. (1966). Contributions to Distribution-Free tests. Approved Ph.D. Thesis, Banaras Hindu University.

Stuart, A. (1952) : Asymptotic relative efficiencies of distribution-free tests of randomness against normal alternatives. J. Amer. Stat. Assn., 47, 49.

Wetherill, G.B. (1960). The Wilcoxon test and the non-null hypothesis. J. Roy, Stat. Soc., **B, 22,** 402-418.

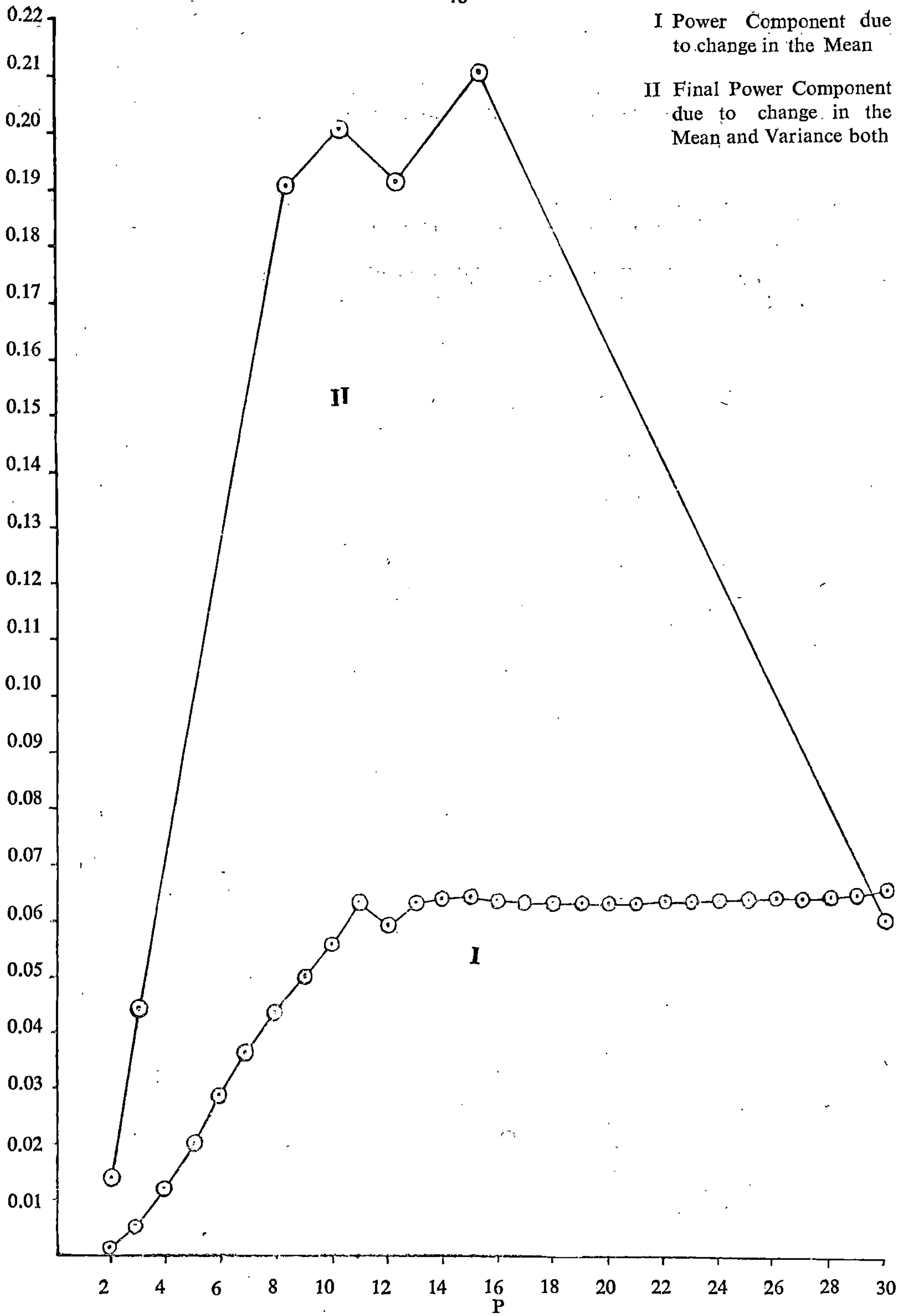

Fig. 1. Relative power for T-Statistics for testing two normal samples of size 15 each

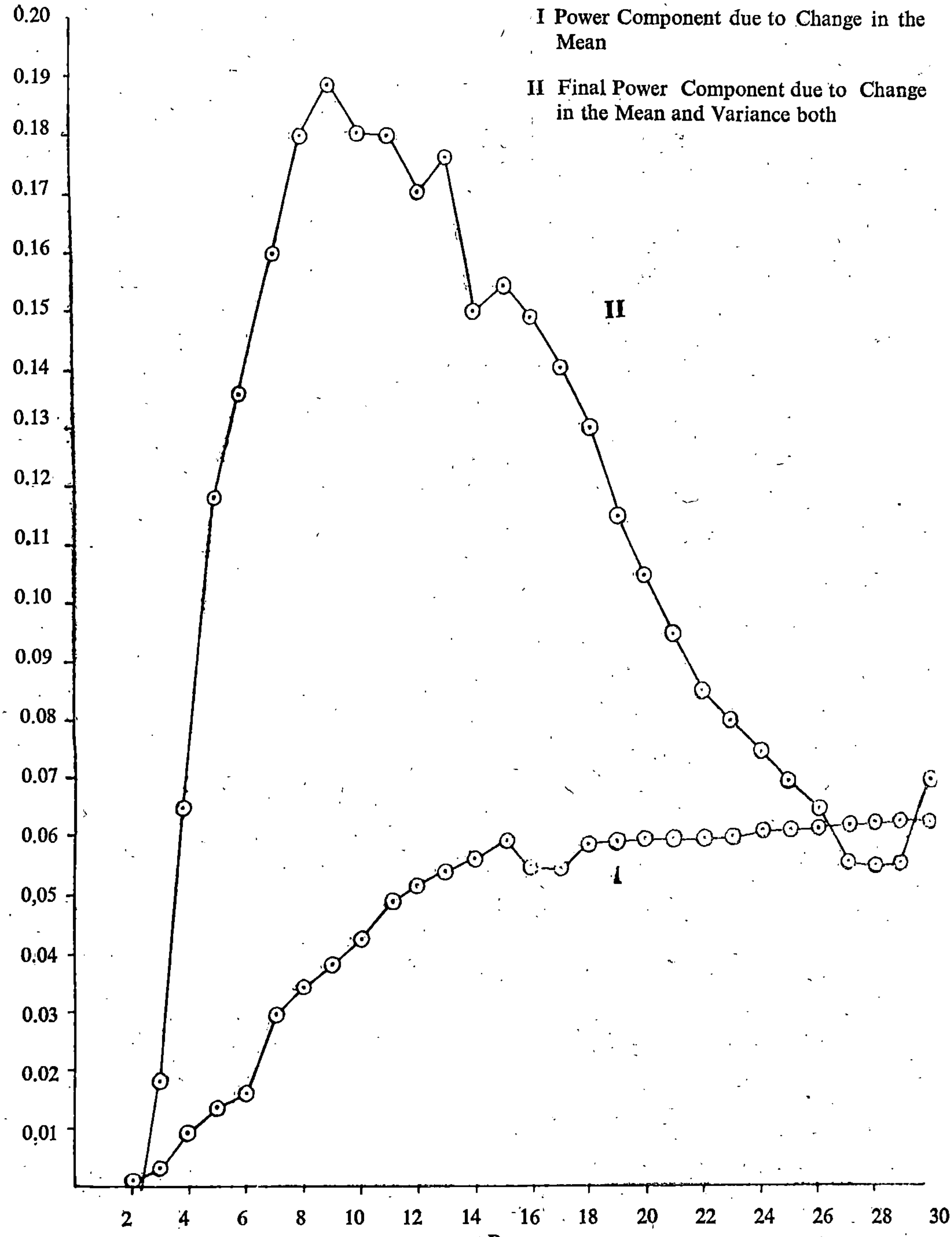

Fig. 2. Power components for T-statistics for testing two normal samples of size 15 on the basis of Monte Carlo studies